FROM QUANTUM THEORY TO MUSIC

MARIA LUISA DALLA CHIARA, ROBERTO GIUNTINI, AND ELEONORA NEGRI

Abstract. Albert Einstein has been, at the same time, one of the "founding fathers" and one of the most serious critics of quantum mechanics. A rigorous criticism of the theory has been developed in a celebrated article, jointly written with Podolsky and Rosen, where one of the most famous quantum theoretic paradoxes (in jargon termed "EPR-paradox") has been presented for the first time. There is something paradoxical in the history of this paradox. Most of the features that Einstein Podolsky and Rosen had described as negative consequences of quantum theory have been, later on, converted into theoretic and practical advantages, even from the technological point of view. We need only think that *teleportation*, *quantum computation* and *quantum cryptography* systematically use, in a positive way, some characteristic quantum phenomena that are usually called *EPR-situations*. We are now beginning to understand how the mysterious *quantum entanglement* (which represents the most intriguing feature of the EPR-situations) can be also applied to a formal analysis of some characteristic semantic phenomena, where *holistic* and *contextual* aspects play a relevant role. To what extent is it reasonable and interesting to apply some general ideas that arise in the "far" quantum world in order to reconstruct formally the deep semantic structures that underly musical compositions? We try and give some answers to this question.

1. Einstein and the "musicality" of quantum mechanics

Albert Einstein has once observed that there is *something musical* in the structure of quantum mechanics. What did the great physicist, who liked to play violin, really mean by this somewhat mysterious remark?

Einstein's attitude towards quantum theory (QT) has been, in a sense, paradoxical. In fact, Einstein has been at the same time, one of the "founding fathers" and one of the most serious critics of the theory. While his idea about the *musicality* of QT is not well known, one often quote his celebrated claim "I cannot believe that God is playing dice".

A rigorous criticism of QT has been developed in a celebrated article, jointly written with Podolsky and Rosen, "Can quantum mechanical description of reality be considered complete?" This is the paper where one of the most famous quantum theoretic paradoxes (in jargon termed "EPR-paradox") has been presented for the first time. There is something paradoxical in the history of this paradox. Most of the features that Einstein

---

Key words and phrases. quantum theory, quantum computation, music.

Podolsky and Rosen had described as negative consequences of QT have been, later on, converted into theoretic and practical advantages, even from the technological point of view. We need only think that *teleportation*, *quantum computation* and *quantum cryptography* systematically use, in a positive way, some characteristic quantum phenomena that are usually called *EPR-situations*.

Beyond the intentions of the authors, the EPR-article has stimulated a genuine "epistemological revolution", forcing us to discuss, according to new modalities, some crucial problems of the theory of knowledge: the relationship between an *observing subject* and an *observed object*, the notions of *physical object* and of *physical property*, the relation *whole-parts* for physical objects, the *space-time framework* and the role of *probability*.

We are now beginning to understand how the mysterious *quantum entanglement* (which represents the most intriguing feature of the EPR-situations) can be also applied to a formal analysis of some characteristic semantic phenomena, where *holistic* and *contextual* aspects play a relevant role.

As is well known, the traditional semantic theories, based on classical logic, are "desperately" *analytical* and *antiholistic*. A basic principle in these theories is a *compositionality-assumption*, according to which the meaning of any compound expression is determined by the meanings of its parts. Furthermore, meanings are always supposed to be *precise* and *non-ambiguous*. As a consequence, classical semantics turns out to be hardly applicable to an adequate formal analysis either of natural languages or of the languages of art, where contextuality and ambiguity seem to represent essential features. The quantum-theoretic formalism, instead, gives rise to some characteristic *entangled states of knowledge* where our information about the *whole* determines our information about the *parts*. And the procedure cannot be, generally, inverted: in other words, it is impossible to reconstruct the global information as a mere combination of the partial pieces of information about the component elements.

To what extent is it reasonable and interesting to apply some general ideas that arise in the "far" quantum world in order to reconstruct formally the deep semantic structures that underly musical compositions? We will try and give some answers to this question.

2. The EPR-paradox

Most of the logical difficulties of QT depend on an unsolved conflict between two basic postulates of the theory: *Schrödinger-equation* and *von Neumann's collapse of the wave function*. Consider a particle (say an electron) evolving during a time-interval $[t_0, t_1]$ and suppose that, at the initial time t_0, an observer has associated to our particle a given *state*, which sums up his (her) information about the object under investigation. In such a case, Schrödinger-equation determines a unique "history", which can be represented as a sequence of states of our particle.

What is exactly a *state* of a given particle? In the happiest situation, a state may correspond to a *non-contradictory maximal knowledge*. This means that one is dealing with a knowledge that cannot be consistently extended to a more precise information (in the language of the theory). Even a hypothetical omniscient mind could not know more. Pieces of information of this kind are usually called, both in classical and in quantum physics, *pure states*. States representing non-maximal knowledge are instead called *mixtures* (or *mixed states*). In the mathematical formalism of quantum theory, both pure and mixed states are identified with some special kinds of abstract objects, that "live" in an abstract space representing the "mathematical environment" for the quantum system under investigation. Technically, these spaces are called *Hilbert spaces* (special examples of vector spaces based on the set of all complex numbers). In this framework, pure states (usually indicated by $|\psi\rangle$, $|\varphi\rangle$, ...) are particular vectors that "live" in a convenient Hilbert space $\mathcal{H}$.

Unlike classical pure states, quantum states are essentially *logically incomplete*. For, a pure state $|\psi\rangle$ does not *semantically decide* all the physical properties that may hold for the system described by $|\psi\rangle$. As a consequence of Heisenberg's *uncertainty principle*, many properties are necessarily *indeterminate*.

On this basis, the *history* of a given particle during a time-interval $[t_0, t_1]$ can be represented as a sequence of states (which may be either pure or mixed):

$$(s(t_0), \ldots, s(t_1)),$$

where $s(t_0)$ represents the state at the initial time t_0, while $s(t_1)$ is the state at the final time t_1.

Consider now a property P that may hold for the particle under investigation. For instance, the particle might be an electron and P might represent the property "the value for the quantity *spin* in the x direction is *up*". Suppose that during the interval $[t_0, t_1]$ the observer chooses to test whether the particle satisfies the property P that is undecided for the initial state $s(t_0)$. Then, soon after the measurement, the state $s(t_0)$ shall be *reduced by collapse of the wave function* to a new state $s^*(t_1)$, which decides whether either P or its negation *not-P* holds. All this gives rise to new history:

$$(s^*(t_0), \ldots, s^*(t_1)),$$

where each element is determined by the final state $s^*(t_1)$ and by Schrödinger-equation applied to this state (from the present time towards the past). One obtains, in this way, a plurality of "histories", which is not easy to interpret from an intuitive point of view.

We will now try and illustrate the EPR-paradox in a simple way, using a metaphorical description (which will preserve the logical structure of the argument). The paradox refers to a particular physical situation that involves two quantum particles (say, two electrons), which will be called *Sarah* and

Susan in the framework of our metaphor. We suppose that the two objects have interacted in the past and are separated since time t_0. One is dealing with a "very strong" (*space-like*) separation: in other words, *Sarah* and *Susan* cannot exchange any signal during the time-interval we are referring to. Two observers *Oswald* and *Oscar* are observing *Sarah* and *Susan*, respectively. We are interested in a pair of *incompatible physical quantities*, indicated by $\mathcal{P}$ and $\mathcal{Q}$, that cannot be simultaneously measured, by Heisenberg's *uncertainty principle* (for instance, the quantity *spin* in the x direction and the quantity *spin* in the y direction). In our metaphor, we can imagine that $\mathcal{P}$ represents the hair-color, while $\mathcal{Q}$ represents the eyes-color. Both quantities $\mathcal{P}$ and $\mathcal{Q}$ can only assume two possible values: + (positive) and − (negative). As a consequence, we obtain two pairs of possible properties

$$(P^+, P^-) \text{ and } (Q^+, Q^-)$$

(say, (*dark hair*, *light hair*) and (*dark eyes*, *light eyes*)).

Owing to the past interaction, there is a strong correlation between *Sarah* and *Susan*, which concerns the values of the quantities under consideration:

Sarah (Susan) verifies P^+ [*has dark hair*]
if and only if
Susan (Sarah) verifies P^- [*has light hair*].

A similar relation holds for the pair Q^+, Q^- (dark eyes/light eyes).

Suppose that at the initial time t_0 the two quantities $\mathcal{P}$ and $\mathcal{Q}$ are totally indeterminate both for *Sarah* and for *Susan*. Soon before time t_2 (in the interval $[t_1, t_2]$) *Oswald* decides to measure quantity $\mathcal{P}$, discovering that

Sarah is P^+ [*has dark hair*].

In the metaphor, *Oswald* might take *Sarah*'s hat off, discovering that she has dark hair (Figure 1).

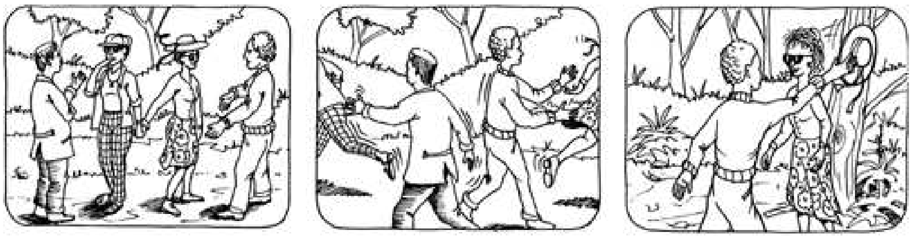


FIGURE 1

Hence, owing to the correlation between *Sarah* and *Susan*, *Oswald* knows, without having in any way interacted with *Susan*, that

Susan is P^- [*has light hair*].

At the same time, we suppose that the other observer *Oscar* did not perform any measurement (he did not take either *Susan*'s hat or *Susan*'s glasses off). A question arises: how to determine *Sarah*'s and *Susan*'s "true histories" during the time-interval $[t_0, t_2]$? For *Sarah*, the most natural answer seems to be the following: her history is determined by the state-sequence

$$(s^a(t_0), s^a(t_1), s^a(t_2)),$$

where $s^a(t_1)$ is obtained from $s^a(t_0)$ by application of Schrödinger-equation, while $s^a(t_2)$ is obtained by collapse of the wave function. Property P^+, which is indeterminate for states $s^a(t_0)$ and $s^a(t_1)$, is decided for state $s^a(t_2)$ (because *Oswald* has discovered that *Sarah* has dark hair). What about *Susan*? At first sight, two different histories seem to be legitimate. The first one is the history that is "seen" by *Oscar*, who did not perform any measurement:

$$(s^u(t_0), s^u(t_1), s^u(t_2)),$$

where all states are determined by Schrödinger-equation and where in any time properties P^+ and e P^- are undecided. The second one is the history that is "imagined" by *Oswald*, who (having measured quantity $\mathcal{P}$ on *Sarah*) knows that at time t_2

$$\textit{Susan is } P^- \ [\textit{has light hair}].$$

As a consequence, the history imagined by *Oswald* will be the following:

$$(s^u(t_0), s^u(t_1), s^{u^*}(t_2)),$$

where the final state $s^{u^*}(t_2)$, obtained by collapse of the wave function, decides that *Susan* is P^- (has light hair). Is it reasonable to ask (as we sometimes do in real life): which is the "true history" of *Susan*?

The concept of "true history" turns out to be deeply connected with the critical *reality-principle*, which represents the main philosophical hypothesis of the EPR-argument. The principle is formulated as follows:

> *If, without in any way disturbing the system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of reality corresponding to that physical quantity.*

Apparently, the reality principle proposes a *sufficient condition* for a physical property to be considered *objective* (i.e. independent of any observer's action). Let us ask: does P^- (having light hair) represent an objective property for *Susan*? A positive answer to this question seems to be justified by the reality-principle combined with a *locality-hypothesis*, according to which *no superluminal influence* is admitted. *Susan* cannot be disturbed by any action performed by a far observer like *Oswald* (by the locality-principle). As a consequence, by the reality-principle, P^- represents an *objective property* for *Susan*.

We can now use a *counterfactual argument* (based on an implication whose antecedent is false). During the time interval $[t_1, t_2]$, *Oswald* might have chosen to measure quantity $\mathcal{Q}$ instead of quantity $\mathcal{P}$. As a consequence, the following counterfactual implication must be correct:

Should Oswald have chosen to measure $\mathcal{Q}$, then either Q^+ or Q^- would be an objective property for Susan at time t_2.

However, by definition of objectivity, *Susan*'s objective properties cannot be determined by *Oswald*'s choices (which depend on his free will). Hence, we obtain:

Either $[P^-$ *and* $Q^+]$
or
$[P^-$ *and* $Q^-]$
is an objective property for Susan at time t_2.

This conclusion is justified by the objective character of property P^- and by a natural logical principle according to which *the conjunction of two objective properties is an objective property*. At this point, the argument invokes another important hypothesis, the *physical completeness* of QT, formulated by Einstein, Podolsky and Rosen as follows:

> *Every element of physical reality must have a counterpart in the physical theory.*

In other words: if the theory is physically complete, then all objective properties (expressed in the language of the theory) should be reflected by some corresponding pure states.

Hence, in particular, there should be a pure state of *Susan* that assigns probability 1 to one of the two following properties:

1) $[P^-$ *and* $Q^+]$ [*light hair and dark eyes*];
2) $[P^-$ *and* $Q^-]$ [*light hair and light eyes*].

However this result contradicts the uncertainty principle: the two quantities $\mathcal{P}$ and $\mathcal{Q}$ are incompatible; consequently, no state can assign a precise value to both of them. We have obtained, in this way, a formal contradiction that seems to put in question the logical soundness of quantum mechanics.

Is it possible to block the derivation of such a paradox? The proof of a contradiction in a scientific theory is, in a sense, similar to the discovery of a murder in the framework of a detective story. And each solution that is proposed to avoid the contradiction plays the role of a detective who identifies the murderer. Of course, as happens in detective stories, also scientific paradoxes generally give rise to different possible solutions. In the case of the EPR-argument the possibly "guilty" hypotheses are:

1) The reality-principle;
2) the locality-principle;
3) the physical completeness-principle.

Each solution is characterized by a different choice of some guilty hypotheses. Einstein, Podolsky and Rosen had no doubts: the hypothesis, that has to be refused, is the physical completeness-principle. In fact, the original version of the EPR-argument was presented as a kind of *proof by contradiction* whose conclusion was: *QT is physically incomplete*. In other words, the pure states of the theory do not represent a *maximum of information*; one is dealing with a kind of *statistical pieces of information*, that are quite similar to the states of classical statistical mechanics. The article "Can quantum-mechanical description of reality be considered complete?", concludes as follows:

> *While we have thus proved that the wave function does not provide a complete description of the physical reality, we left open the question of whether or not such a description exists. We believe, however, that such a theory is possible.*

However, this conclusion contains a "logical mistake". In fact, the EPR-argument only proves the logical incompatibility between QT and our three general principles, without forcing us to choose a particular "guilty hypothesis". There are other legitimate solutions, which make different choices. For instance, the solution proposed by Niels Bohr and by the "Copenhagen-school" is based on the refusal of the reality-principle. According to Bohr, it is not sensible to speak of "elements of reality", because *all properties of physical objects have to be described as relations*. In our days, many scholars are in favor of solutions based on a refusal of the locality-principle. In this perspective, the action performed by *Oswald* in the interval $[t_1, t_2]$ turns out to have a "genuine physical influence" on *Susan*, in spite of the space-like separation. However, this result does not imply the possibility of sending a signal from *Oswald* to *Oscar* during the interval $[t_1, t_2]$. Accordingly, there is no conflict between quantum non-locality and special relativity (which was Einstein's basic worry). Is it possible to recover, in this way, the *unicity* of *Susan*'s history? The question does not admit an easy answer.

In 1985, half a century after the appearing of the EPR-article, only Nathan Rosen was still alive. A number of conferences were organized to celebrate the discovery of the famous paradox; and sometimes the "honor-guest" was, of course, Rosen. How did the "third man" of the *terzetto* regard the EPR-argument, fifty years after? The following quotation represents an interesting witnessing:

> *At the time of the writing of the EPR paper I agreed with the belief expressed at the end that a complete theory is possible. Since then fifty years have passed and physics has changed greatly. In recent years doubts have arisen in my mind as to whether a theory will be found in the future that will be complete according to the criteria of the paper and will be correct in giving agreement with observation.. Hence it is hard to believe that a theory will be found that will be complete, based on the criterion of an element of reality, used in the paper. It may also be that in the future physical theories will describe reality in different terms from those to which we are now accustomed. Does this means that the EPR paper is useless? I think not. The paper has led to a great deal of discussion*

> *that has helped to clarify the physical concepts. I like to believe that this has contributed, if in a small measure, to the progress of physics.*[2]

3. THE INTERACTION BETWEEN ACTUAL AND POSSIBLE OBJECTS

The strange behavior of quantum objects has changed some general ideas about the interaction between *what does exist* and *what might exist*. In the framework of classical semantics, *actual* and *possible objects* were sharply distinguished: the set of all *actual objects* was described as a *classical proper subset* (with no *fuzzy* borders) of the class of all possible objects. Quantum logic (created by Birkhoff and von Neumann as a natural abstraction from the mathematical formalism of quantum theory) is, in this respect, "more liberal": actual existences generally depend on virtual existences! As noticed by the philosopher and logician W.O.Quine, the traditional concept of *physical object* tends to "evaporate" in modern physics.

Any logical investigation about the *existence problem* is of course concerned with the semantic analysis of the *existential quantifier* ($\exists$), which behaves differently in different logics. When discussing these questions, it is useful to refer to a successful and flexible kind of semantics that has been termed *possible world semantics.* The basic intuitive idea is very simple and goes back to Leibniz. We suppose that the *actual world* (indicated by W) is *correlated* with a number of *alternative situations*, also called *possible worlds.* Of course, the actual world itself represents an example of a possible world. The correlation between worlds is usually called *accessibility relation.* From an intuitive point of view, a world W_1 is accessible to another world W_2, when W_2 represents a *reasonable alternative* for W_1.

In spite of its metaphysical aspect, the possible world semantics admits a quite natural physical interpretation and can be applied to quantum logic. The basic intuitive idea is the following: any possible world represents the knowledge of an observer about a quantum object (say an electron or a photon). Hence it can be mathematically represented as a possible state of the object under investigation.

What about the quantum meaning of the accessibility relation? Mathematically, two pure states $|\psi\rangle$ and $|\varphi\rangle$ are accessible when they are *non-orthogonal* in the abstract space $\mathcal{H}$, where all pure states "live". This corresponds to the following physical relation: $|\psi\rangle$ is accessible to $|\varphi\rangle$ when $|\psi\rangle$ can be transformed into $|\varphi\rangle$ after the performance of a measurement concerning a *physical quantity* that can be measured on the system. The state-transformation $|\psi\rangle \mapsto |\varphi\rangle$ (induced by the measurement) is determined by *collapse of the wave function.*

In order to understand the physical interpretation of the quantum-logical existential quantifier, let us first recall that in most logics the existential quantifier ($\exists$) is a generalization of the disjunction ($\vee$). As an example, let us think of the original human universe represented by the set whose only elements are *Adam* and *Eve*. In this universe, the existential sentence

There exists somebody who ate the apple

is clearly equivalent to the disjunctive sentence

Either Adam or Eve ate the apple.

Of course, in the case of our present human universe, using disjunctive sentences (with about 6 billions of members) instead of existential sentences would be quite unpractical!

Also in quantum logic, the existential quantifier $\exists_q$ is a generalization of the quantum-logical disjunction $\vee_q$. The truth-condition for disjunctive sentences is the following:

The sentence

Either A ***or*** B

is *TRUE* in the actual world W

$(\models_W A \vee_q B)$

if and only if

for any possible world W_1 accessible to W there exists a possible world W_2 accessible to W_1 such that either A is true in W_2 or B is true in W_2.

Apparently, truth and falsity in the actual world essentially depend on what happens in other possible worlds. As a consequence, a disjunction may be *true* (in the actual world) even if both members are *indeterminate*. Such a semantic behavior, which may appear *prima facie* somewhat strange, seems to reflect pretty well a number of concrete quantum situations. In fact, in quantum theory one is often dealing with alternatives that are semantically determined and true, while both members are, in principle, indeterminate.

4. The two-slit experiment

One of the fundamental experiments of quantum mechanics, the *two slit-experiment*, has been often described as a kind of *experimentum crucis* for quantum logic. One is dealing with an intriguing result that R. Feynman (in his celebrated *Lectures on Physics*) has commented as follows:

> *It is all quite mysterious. And the more you look at it, the more mysterious it seems.*

In order to understand, from an intuitive point of view, why the two-slit experiment appears so "mysterious", we will follow an amusing description that can be found in the book *Alice in Quantum Land. An Allegory of Quantum Physics* by R. Gilmore. During her fantastic travel through the mysteries of the quantum world, Alice arrives at the *Gedanken room*, where all *thought-experiments* become *real.* First the *Classical Mechanic* illustrates the two-slit experiment, using a bullet-gun. His "classical" gun begins firing a number of bullets. Most of them whine off in all directions; but some of them go through a screen with two slits (say slit 1 and slit 2) and finally hit the wall (in front of the screen), each leaving one precise mark. The final configuration of the marks on the wall gives rise to a typical *additive pattern*, representing the *probability* that a generic bullet reaches a given region of the wall (Figure 2).

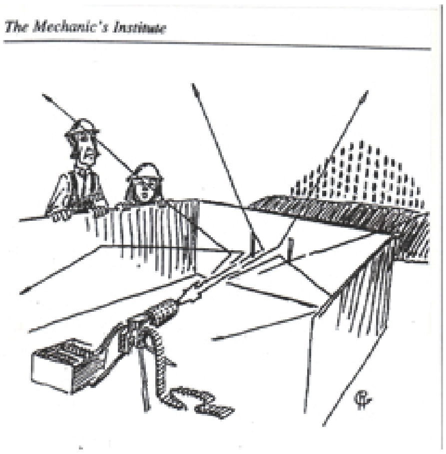


FIGURE 2. From R. Gilmore, *Alice in Quantum Land. An Allegory of Quantum Physics.*

The *Classical Mechanic* explains to Alice:

> "Since both slits are open, the bullets may pass through either slit, so the overall distribution is given by the sum of the probabilities that we got for the two slits on their own, since the bullets must have passed through one or the other. They cannot have passed through both, you know."

Soon after, the *Quantum Mechanic* makes a similar experiment, using an *electron gun.* The effect is completely different: a clear *interference pattern* appears, quite similar to what would be the case in a wave-phenomenon (Figure 3).

The *Quantum Mechanic* explains:

> "There you see a clear *interference effect.* With one slit alone we would have seen that the distribution would decrease smoothly to either side, much as the bullets did. In this case, you see that, when there are two slits open, the *amplitudes* from the two slits are interfering and are producing obvious peaks and troughs in the probability distribution. The behavior of the electrons is quite different from that of my friend's bullets."

Figure 3. From R. Gilmore, *Alice in Quantum Land. An Allegory of Quantum Physics.*

> "I do not understand - says Alice - Do you mean that there are so many electrons going through that somehow the electrons which go through one hole are interfering with the ones which go through the other?"
>
> "No. Not at all - replies the *Quantum Mechanic* - The interference effect works even when there is only one electron present in any time. One electron on its own can show interference. It can go through both slits and interfere with itself."
>
> "But that is silly! - cries Alice - One electron cannot go through both slits. As the *Classical Mechanic* said, it just isn't sensible."

Quickly she directs the light toward the two slits:

> "I can see the electrons as they go through the slits and it is just as I said it must be. Each one does go through just one slit."
>
> "Aha! - replies the *Quantum Mechanic* - but have you looked to see what is happening to the interference pattern? The interference effects only happen when there is no way that you could know which slit the electron went through. Whether or not you do know does not matter. So you see, when there is interference it seems as if each electron is going through both slits. If you try and check on this you will find that the electrons go through one slit, but then the interference vanishes. You can't win!".

Apparently, the case of the interference pattern, arising when no observation is possible, gives rise to a typical quantum-logical behavior of the connective *or*. The disjunction:

Either the electron has gone through slit 1 *or*

the electron has gone through slit 2

is true. At the same time, both members of the disjunction are indeterminate and consequently not true. When, instead, *observing* is possible (even if no *actual observer is observing*), the interference pattern disappears and the connective *or* behaves according to classical logic. The truth of the disjunction

Either the electron has gone through slit 1

or

the electron has gone through slit 2

implies that each electron has gone through a well determined slit.

5. Quantum computers and Gestalt-thinking

Interestingly enough, *quantum computers* give rise to a situation that is quite similar to what happens in the two-slit experiment: the strong *parallelism*, characteristic of quantum computers, is essentially based on *virtual paths*, that a quantum computer can follow at the same time. *Reality is, in a sense, entangled with possibility*.

Quantum computation has recently suggested some new forms of quantum logic (called *quantum computational logics*), where *meanings* of sentences are identified with quantum information quantities. This provides a mathematical formalism for an abstract *theory of meanings* that can be applied to investigate different kinds of semantic phenomena where *holistic*, *contextual* and *gestaltic* patterns play an essential role (from natural languages to musical compositions).

As is well known, human perception like thinking seems to be essentially *synthetic*. We never perceive an object by *scanning* it point by point. We instead form right away a *Gestalt*, i.e. a global idea of it. Rational activity, as well, seems to be essentially based on *gestaltic patterns*.

Gestalt-thinking cannot be adequately represented in the framework of classical semantics, which is basically *analytical* and *compositional*: the meaning of a *compound* expression is always determined by the meanings of its *parts*. At the same time, meanings are *non-ambiguous* and *sharp*.

All this renders classical semantics hardly applicable to an adequate analysis of natural languages and of artistic contexts, where holistic and ambiguous features seem to play a relevant role. As a significant example, we might refer to the final verse of the poem *L'Infinito* by Giacomo Leopardi:

E 'l naufragar m'è dolce in questo mare
(And drowning in this sea is sweet to me).

This verse has been compared with the last words of Isolde in Wagner's *Tristan und Isolde*:

ertrinken, versinken, unbewusst, höchste Lust!

In Leopardi (but, in a similar way, also in Wagner), the poetic result seems to be essentially connected with the following semantic relation: the meanings of the component expressions “naufragar” (drowning), “dolce” (sweet), “mare” (sea) do not correspond here to the most common meanings. By the way, there is no sea in Recanati, Leopardi’s native village which the poem refers to. However the usual meanings of our expressions are somehow present and ambiguously correlated with the metaphorical meanings that are evoked by the whole poem. Needless to say, this represents a quite typical semantic situation in poetry.

6. Quantum computational logics

In the semantics of quantum computational logics the following conditions are satisfied:

1) *global meanings* (which may correspond to a *Gestalt*) are intrinsically *vague*, because they leave semantically undecided many relevant properties of the objects under investigation;

2) any global meaning determines some *partial meanings*, which are generally vaguer than the global one;

3) meanings (*Gestalten*) can be generally represented as *superpositions* of other meanings, possibly associated to probability-values;

4) meanings (in the same way as *Gestalten*), are dealt with as intrinsically dynamic objects.

In this framework, the *meaning* of any sentence is identified with a quantum information quantity: a system of *qubits* or, more generally, a *mixture* of systems of qubits (also called *qumix*).

What is a qubit? From an intuitive point of view, a *qubit* can be regarded as the quantum variant of the classical notion of *bit*. As is well known, in classical information theory, one bit measures the information that is transmitted (or received), whenever one chooses one element from a set consisting of two elements (for instance, from the set consisting of the answer *YES* and of the answer *NO*, or from the set consisting of the number 1 and of the number 0). In quantum information, one cannot generally refer to precise answers (like *YES* or *NO*). The typical answer is represented by a *quantum perhaps*, that can be described as a quantum *superposition* of the answer *YES* and of the answer *NO*.

By using Dirac’s “ket-notation” in a non-technical fashion, we can write the general form of a qubit as follows:

$$|QUBIT\rangle = |NO_a\rangle \ + \ |YES_b\rangle$$

where:

- a is a (complex) number that determines the probability of *NO*,
- b is a (complex) number that determines the probability of *YES*.

The numbers a and b are usually called *quantum amplitudes.*

From the physical point of view, a qubit can be regarded as the *pure state* of a single particle, while a system of n qubits (also called *quregister*) corresponds to the state of a compound system consisting of n particles. The idea is that a single particle (like an electron) can physically carry the information-quantity represented by one qubit. In order to carry the information stocked by n qubits we need, of course, a compound system consisting of n particles.

From the mathematical point of view, qubits are particular vectors that "live" in a two-dimensional Hilbert space. Hence, the mathematical form of a qubit is usually written as follows:

$$|\psi\rangle = a|0\rangle + b|1\rangle,$$

where $|\psi\rangle$, $|0\rangle$ and $|1\rangle$ are vectors (whose length is 1) in a two-dimensional Hilbert space. From an intuitive point of view, the vector $|0\rangle$ represents the answer *NO*, while the vector $|1\rangle$ represents the answer *YES*.

What about systems of n qubits (n-quregisters)? As we have seen, an n-quregister is a possible state of a compound system, consisting of n particles. The mathematical environment for such a system can be represented as a special product (called *tensor product*) of n two-dimensional Hilbert spaces. On this basis, the mathematical representative of an n-quregister is identified with a unit-vector of such a product space (whose dimension depends on n).

We will now sketch the basic intuitive idea of the quantum computational semantics. The starting point is represented by the following assumption: a knowledge stored by a qumix can be linguistically expressed by a sentence of an appropriate language $\mathcal{L}$. Accordingly, any *semantic interpretation* of $\mathcal{L}$ will associate to any sentence α a qumix ρ_α that represents its *informational meaning*:

$$\alpha \;\mapsto\; \rho_\alpha .$$

At the same time, the logical connectives of the language (like *not*, *and*, *or*,...) are interpreted as *quantum logical gates*: special operations (called *unitary quantum operations*) that transform qumixes into qumixes in a reversible way.

The qumix ρ_α (representing the meaning of α) "lives" in a Hilbert space whose dimension depends on the logical form of α. The simplest examples of sentences are *atomic* sentences, that cannot be decomposed into more elementary sentences (like, for instance, "7 is odd"). Accordingly, the meaning of such sentences "live" in the simplest Hilbert space: the two-dimensional space (where all qubits are located). A molecular sentence whit n occurrences of atomic sentences (for instance, "7 is odd *and* 8 is even", where $n = 2$) can be regarded as a linguistic description of a compound physical system consisting of n particles. In fact, we need n particles in order to carry the information that is expressed by our molecular sentence. On this basis, it is natural to assume that the meaning of such a sentence "lives" in a product-space, whose dimension depends on n. Thus, the logical complexity of a sentence is *measured* by the number of atomic sentences occurring in it. And any sentence determines a particular Hilbert space, that represents its *semantic environment*.

As we have seen, the qumix ρ_α (representing the meaning of the sentence α) corresponds to a possible state of a physical system (carrying the information expressed by α). In this framework, it is quite natural to apply the statistical rules of quantum theory that permit us to calculate for any state and for any physical property the *probability that a physical system in that state satisfies that property*. As expected the kind of property that turns out to be relevant for informational aims is represented by the *truth-property*, which can be formally dealt with as a "normal" physical property (like "moving at light velocity" or "being located in a given space-region"). On this basis, any qumix ρ has a well determined probability value $prob(\rho)$. From an intuitive point of view, the number $prob(\rho)$ represents *the probability that the information stored by ρ is true*. As an example, suppose we are in the simplest situation where our qumix ρ is a single qubit, having the standard form:

$$|\psi\rangle = a|0\rangle + b|1\rangle.$$

In such a case, the probability that the information stored by $|\psi\rangle$ is true is determined by the amplitude b (the "companion" of the bit $|1\rangle$, representing the answer *YES*). More precisely, $prob(\rho)$ is identified with the number $|b|^2$ (the squared modulus of b).

7. Quantum computations and quantum epistemic trees

How to represent *quantum computations*? As we have seen, the general equation that describes the dynamic evolution of quantum systems is the *Schrödinger-equation* (which determines how the state $|\psi(t_0)\rangle$ of a quantum system at an initial time t_0 evolves to another state $|\psi(t_1)\rangle$ at a final time t_1). The general form of the equation is the following:

$$|\psi(t_0)\rangle \quad \mapsto \quad |\psi(t_1)\rangle = U_{[t_0,t_1]}|\psi(t_0)\rangle,$$

where $U_{[t_0,t_1]}$ is a *unitary operator*, representing a *reversible transformation*. Since one is dealing with a reversible process, one can go back and forth without any dissipation of information.

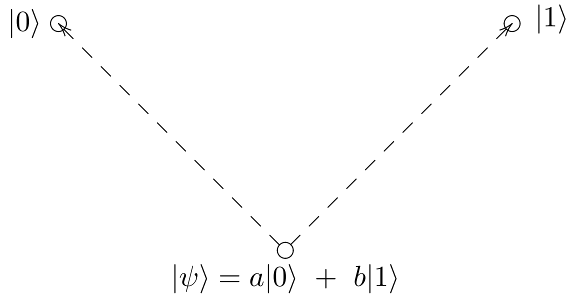
The Schrödinger-equation pattern can be naturally applied to quantum information. Qumixes are nothing but states of quantum systems, while quantum logical gates are unitary operations processing information in a reversible way. In other words, *quantum states of knowledge* are transformed into quantum states of knowledge, without any dissipation of information. From an intuitive point of view, such a process can be naturally represented by a *quantum epistemic tree* from an initial state of knowledge $\rho(t_o)$ to a final state of knowledge $\rho(t_1)$:

- $\rho(t_1)$

  $\Uparrow$
- ....

  $\Uparrow$
- $\rho(t_0)$

Although the superficial form of a quantum epistemic tree is that of a *linear process*, the deep structure is essentially *parallel*. For, any qubit $|\psi\rangle$ generally gives rise to a *branching*:



In other words, a state of knowledge represented by a quantum superposition $|\psi\rangle$ reflects, at the same time, two parallel epistemic paths: the first one leads to the answer *YES*, while the second one leads to the answer *NO*. From an intuitive point of view, *quantum computations* can be regarded as characteristic examples of quantum epistemic trees.

8. Entangled information and holistic semantics

One of the most intriguing aspects of quantum theory concerns *entanglement-phenomena*. What is exactly entanglement? From an intuitive point of view the basic features of an *entangled state* $|\psi\rangle$ can be sketched as follows:

- $|\psi\rangle$ is a maximal information (a pure state) that describes a compound physical system S;
- the information determined by $|\psi\rangle$ about the parts of S is non-maximal. Hence, the states of the whole system is a pure state, while the states of the parts (which are determined by the state of the whole) are proper mixtures.

Entangled states play an essential role in all *EPR-situations*. For instance, in the case of our metaphorical description of the EPR-paradox, *Sarah* and *Susan* are typically entangled. Before *Oswald*'s measurement, the state of the compound system (*Sarah + Susan*) is a pure state that assigns probability $\frac{1}{2}$ to the two following events:

- *Sarah has dark hair and Susan has light hair*;
- *Sarah has light hair and Susan has dark hair*.

A maximal information about the compound system (*Sarah* + *Susan*) determines two partial pieces of information about the component parts (*Sarah*, *Susan*). However, such partial pieces of information cannot be represented by pure states. As a consequence, the two individual histories of *Sarah* and of *Susan* turn out to be necessarily ambiguous, before *Oswald*'s measurement.

Entanglement-phenomena can be naturally used to model some typical holistic semantic situations in the framework of our quantum computational semantics.

We can consider *entangled states of knowledge*, represented by special quregisters that are meanings of molecular sentences. As an example, consider a conjunctive sentence having the form

$$C = A \textit{ and } B.$$

The following situation is possible:

- the *meaning* ρ_C of the conjunction C is a quregister, which represents a maximal information (a pure state);
- the meanings of both parts (A,B) are quantum-entangled and cannot be represented by two pure states (two quregisters).

We can say that the *sharp meaning* of the conjunction A *and* B determines two *ambiguous meanings* for the parts (A, B), which are represented by two mixed states. In other words, *the meaning of the whole determines the meanings of the parts, but not the other way around*. In fact, one cannot go back from the two ambiguous meanings of the parts to the quregister representing the meaning of the whole. The mixed state representing the ambiguous meaning of A (of B) can be regarded as a kind of *contextual meaning* of A (of B), determined by the *global context*, which corresponds to the quregister $\rho_{(A \textit{ and } B)}$ (the meaning of the conjunction A *and* B).

The quantum computational semantics is strongly Hilbert-space dependent. As a consequence, applications to fields (far from the quantum world) where Hilbert spaces do not play any significant role, seem to be somewhat unnatural. However, by abstracting from the Hilbert-space formalism, one can develop an abstract version of quantum holistic semantics that is Hilbert-space free[1]. In this framework, quregisters and qumixes (representing maximal and non-maximal pieces of information, respectively) are dealt with as special kinds of *intensional objects* with growing complexity, which reflects the logical form of possible sentences. Accordingly, an abstract notion of *reduced information* permits one to define *contextual meanings* in an appropriate way (like in the concrete quantum case). This abstract quantum-like semantics seems to represent a flexible tool that might be naturally applied to a number of different fields (including a formal analysis of natural languages and of the languages of art).

---

[1] See [8].

As an example, we can try and apply such a semantic analysis to the poem *L' Infinito*. Let us (artificially) decompose the poem into the two following sentences:

- B = *'l naufragar m'è dolce in questo mare (drowning in this sea is sweet to me)*
- A = the poem *L' Infinito* without the last verse.

Thus, we obtain:

$$L'Infinito = A \ and \ B.$$

Our semantics describes how the contextual meaning of the last verse B (an ambiguous meaning) is determined by the global meaning of the whole poem.

Of course, a similar semantic analysis can be also applied to the case of musical compositions, where meanings have an intrinsic holistic, contextual and ambiguous behavior.

.

9. A QUANTUM-LIKE HOLISTIC SEMANTICS FOR MUSICAL SCORES

Musical scores are very complicated examples of symbolic languages. It is interesting to analyze how information is coded by musical scores in comparison with the standard formal languages that are used for scientific theories. The most important differences seem to be the following:

- Formal scientific languages are basically *linear* and *compositional*: *words* and *well-formed expressions* are represented as *strings* consisting of symbols that belong to a well determined alphabet.
- Scores, instead, are two-dimensional syntactical objects, which have at the same time a "horizontal" and a "vertical" component. Any attempt to linearize a score would lead to totally counter-intuitive results.

From a semantic point of view, the characteristic two-dimensionality of musical notation seems to be significantly connected with the deep parallel structures that have an essential role in our perception and intellectual elaboration of musical experiences. We might recall, in this connection, a happy metaphor proposed by Antonio Damasio, according to which human brain seems to work like an orchestra!

As is well known, music and spoken talks are usually perceived according to different modalities. While a simultaneous superposition of talks generally determines a psychological sense of uneasiness, music instead gives rise to the mysterious "polyphonic pleasure". As an example, let us think of many duets (or terzettos or quartets) of lyric operas. Generally, a listener perceives the global polyphonic result; at the same time he (she) is able to grasp the different melodic lines as well as the different "thoughts" of all characters who are singing. We might refer to a number of celebrated examples. A significant case is represented by a fragment of a famous duet in the opera *La Traviata*, where Alfredo's father (Germont) tries to convince Violetta that she should leave Alfredo. At the very beginning, Violetta proposes a kind of compromise: "Ah comprendo, dovrò per alcun tempo da Alfredo allontanarmi" ("I understand, I shall stop seeing Alfredo for a certain time..."). This is what she says, by means of a kind of *recitativo*. However, Violetta has understood very well that what Germont is asking for is much more serious: the separation shall be very for ever. From the musical point of view, her thoughts and her anguish are not realized by the melodic line of her singing but rather by the dramatic phrases performed by the strings in the orchestra. And the "contradiction" between what Violetta is saying and what Violetta is thinking is significantly expressed by some dissonant chords (for instance, Violetta "says" an a flat and "thinks" an a).

Is it possible (and interesting) to represent a musical score as a peculiar example of a *formal language*? In a sense, are scores *formalizable*? One can positively answer to this question, by introducing the notion of *formal representation of a musical score*[2]. The basic idea can be sketched as follows. The characteristic two-dimensionality of the musical syntax can be mathematically represented via some abstract matrix-like structures. All score-measures can be formally described as two-dimensional configurations having the following form:

$$\begin{pmatrix} Instr_1 : A_{11} \ldots A_{1n} \\ Instr_2 : A_{21} \ldots A_{2n} \\ \ldots \\ Instr_m : A_{m1} \ldots A_{mn} \end{pmatrix}$$

Each *row* corresponds here to what shall be performed by a particular instrument (say, the first violin); while columns correspond to what shall be played at the same time. As a consequence, the whole score can be represented as a sequence of matrix-like structures, where *horizontal* and *vertical* syntactical combinations coexist. In this framework, the *musical phrases* occurring in the score (which behave like *well-formed expressions* of a formal language) can be identified with special fragments of the score in question.

Let us now turn to semantic problems. How to describe the *possible meanings* of the musical phrases that occur in a given score? As is well known, the sound-world is a typically *relational world* (quite differently from the color-world). Generally, one cannot associate a well-determined meaning to a single note (or to a single sound). In a sense, single notes appear to be all semantically equivalent. The meaning of a single note, of a chord, of a musical phrase is always determined by the *context*. There is no doubt that music requires a holistic and contextual semantics. In this perspective, we can try and apply to music the basic ideas of our (abstract) quantum-like holistic semantics. Following the quantum-example, we can introduce

---

[2] For technical details see [6].

the notion of *semantic interpretation* of a (formal) score **S**. This can be described as a map that assigns to any syntactical musical phrase A of **S** a *meaning* ρ_A, representing a *semantic musical phrase.*

But what are exactly *semantic musical phrases*? In the framework of an abstract semantics, the notion of *semantic musical phrase* can be dealt with as a special kind of *intensional object*, which reflects the linguistic form of a corresponding syntactical phrase (in the same way as qumixes reflect the logical complexity of corresponding sentences of the quantum computational language). On this basis, *contextual musical meanings* can be formally dealt with like in the quantum case. Accordingly, the *meaning of a global musical phrase* will determine the *partial meanings of its parts*, which are generally more ambiguous than the meaning of the whole phrase.

Of course, a semantic interpretation of a score represents a purely *ideal structure*, that should not be confused with a physical performance of the score in question. One is dealing with a kind of *vague invariant* that underlies a number of different historical performances. Significantly enough, we use to speak, for instance, of "Abbado's interpretation of Beethoven's fifth symphony", without necessarily referring to a given historical performance.

A number of intriguing problems in the semantics of music might be successfully analyzed according to the rules of our abstract quantum-like semantics[3]. For instance, a crucial question (which has been deeply discussed by musicologists) concerns the relationship between a given text (say, a poem or a *libretto*) and a musical realization of the text in question. Allegedly, music transforms a given text into a completely new *global semantic object*. There is an interesting test we can perform: what happens when one and the same poem has been set to music by different composers in different times?

As a significant example, we might refer to the celebrated *Lieder* of *Mignon* and *der Harfner* from Goethe' s *Wilhelm Meister*. Consider, for instance, the famous *Lied* "Kennst du das Land" and compare the musical realizations of Schubert (1815) and of Schumann (1849). They are deeply different. Although both of them express the enigmatic and mysterious features of the figure of Mignon, in Schubert there is "something consoling" (for instance, the *incipit* may sound as a kind of quiet lullaby). Schumann's *Lied* instead is totally dominated by a sense of anguish and of tragedy, which are musically expressed by a number of dissonant chords. Is it reasonable to ask: which musical realization is more faithful to Goethe's poem? The answer to this question is, of course, negative. In a sense, each musical realization determines a new poem. And going back from the musical *Lied* to the original poem represents a semantic operation that can be adequately described in terms of our abstract notion of *contextual meaning*.

Following the example of scientific theories, one can reasonably claim that, from an abstract point of view, a musical composition (say, Beethoven's fifth

---

[3] A technical version of this semantics will be presented elsewhere.

symphony) can be represented as a pair

$$(\mathbf{S}, \mathbf{K}),$$

consisting of a (formalized) score and of the class $\mathbf{K}$ of all possible *semantic interpretations* of $\mathbf{S}$ (in the sense of our abstract holistic semantics). Of course, musicians and musicologists are not generally concerned with the class of *all possible* interpretations, but rather with those interpretations that have been actually realized. In this semantic perspective, the history of a given musical work might be described as a kind of "ideal travel" through the class $\mathbf{K}$.

References

[1] A. Einstein, B. Podolsky, N. Rosen , "Can quantum-mechanical description of reality be considered complete?", *Physical Review*, **47** (1935), 777-780.

[2] N. Rosen, "Quantum mechanics and reality", in P. Lahti and P. Mittelstaedt (eds.), *Symposium on the foundations of modern physics*, World Scientific, Singapore, 1985.

[3] M. L. Dalla Chiara, R. Giuntini and R. Leporini, "Quantum Computational Logics. A Survey", in V. Hendricks and J. Malinowski (eds.), *Trends in Logic. 50 Years of Studia Logica*, Kluwer, 2003, 229–271.

[4] M. L. Dalla Chiara, R. Giuntini and R. Leporini, "Logics from quantum computation", *International Journal of Quantum Information*, **3** (2005), 293–337.

[5] M. L. Dalla Chiara, R. Giuntini and R. Leporini, "Compositional and holistic quantum computational semantics", *Natural Computing*, **6** (2007), 113–132.

[6] M. L. Dalla Chiara and R. Giuntini, "A formal analysis of musical scores", *Mathematica Slovaca*, **56** (2006), 591–609.

[7] M. L. Dalla Chiara, R. Giuntini and R. Greechie, *Reasoning in Quantum Theory*, Kluwer, Dordrecht, 2004.

[8] M. L. Dalla Chiara, R. Giuntini and R. Leporini, G. Toraldo di Francia "Quantum computational logics and possible applications", *International Journal of Theoretical Physics*, to appear.

[9] M. L. Dalla Chiara, R. Giuntini, and G. Toraldo di Francia, "Holistic quantum computational semantics and Gestalt-thinking", in A. Bassi, D. Dürr, T. Weber, N. Zanghì (eds.), *Quantum Mechanics. Are there quantum jumps? On the present status of quantum mechanics*, AIP, 2006, Melville, New York, 86–100.

[10] S. Gudder, "Quantum computational logic", *International Journal of Theoretical Physics* **42** (2003), 39–47.

[11] R. Gilmore, *Alice in Quantum Land. An Allegory of Quantum Physics*, Sigma Press, Wilmslow, 1994.

[12] J. van Bendegem, "The relevance of logical tools for (understanding) the musical process", preprint.

(M. L. Dalla Chiara) Dipartimento di Filosofia, Università di Firenze, via Bolognese 52, I-50139 Firenze, Italy
E-mail address: `dallachiara@unifi.it`

(R. Giuntini) Dipartimento di Scienze Pedagogiche e Filosofiche, Università di Cagliari, via Is Mirrionis 1, I-09123 Cagliari, Italy
E-mail address: `giuntini@unica.it`

(E. Negri) Dipartimento di Filosofia, Università di Firenze, via Bolognese 52, I-50139 Firenze, Italy
E-mail address: `eleonora.negri@unifi.it`