Excluded Middle and Bivalence: Lukasiewicz and Lesnievski in comparison

[Draft paper]

Alessandro Becchi
PhD Student
Department of Philosophy
Florence University
notung@unifi.it

In the context of the so-called «First Polish School» (about 1910-1920) a certain standard distinction between *ontological* and *logical* principles was widely accepted. In those years, and in connection with such a distinction, a lively debate took place among some of the most representative figures of that school, with regard to the grounds and the validity of the principle of the Exclude Middle. One of the outcomes of such a debate was the refusal of the (meta-logical) principle of Bivalence and the elaboration of the first many-valued system for the propositional calculus.

In the following, I will try to sketch (i) the criticisms of the logical principle of the Exclude Middle by Lesniewski and Lukasiewicz, on the background of the just mentioned distinction, and (ii) the philosophical motivations which led Lukasiewicz to getting rid of the principle of Bivalence also. In so doing, I hope it will come clear (iii) a certain distinction between the principle of the Exclude Middle and the principle of Bivalence.[1]

§ 1. Ontological and logical principles

According to Lesniewski [1913b] metaphysics is to be understood as «the system of true propositions concerning all objects in general» (p.48), whereas logic is «the discipline which investigates which propositions are true and which are false» (p.49). In other words -and for the sake of perspicuity- metaphysics was conceived as a 'general theory of objects',[2] and many efforts were made towards the elaboration of a 'scientific metaphysics' in this sense (for example, by Lukasiewicz in his early work on the concept of cause[3]). The core of logic is instead placed in the

---

[1] To this aim, I will substantially rely upon the following papers by the two authors:
Stanislaw Lesniewski [1913b], *The critique of the logical principle of the excluded middle*, in: *Collected Works*, Vol. I, Kluwer, Dordrecht (1992), pp. 47-85.
Jan Lukasiewicz [1910b], *The principle of contradiction in Aristotle*, «Review of metaphysics», 24, (1970-71), pp. 485-509.
Jan Lukasiewicz [1910c], *On the principle of the excluded middle*, «History and philosophy of logic», 8, (1987), pp. 67-69.
Jan Lukasiewicz [1922], *On determinism*, in: *Selected Works*, North-Holland P.C., Amsterdam (1970), pp. 110-128.
In organizing some of the topics touched in this paper I am in debt to two illuminating papers by Dr. Arianna Betti: *Lukasiewicz and Lesniewski on Contradiction*, forthcoming in: M. Baghramian& P. M. Simons (eds.), «Lukasiewicz and Modern Logic», Dordrecht, Kluwer Academic Publishers; and *The Incomplete Story of Lukasiewicz and Bivalence*, in T. Childers & O. Majer (eds.), «The Logica Yearbook 2001», Praha, Filosofia, 2002, pp. 21-36. Both papers are available for downloading on the author's web-page: http://www.fmag.unict.it/PolPhil/Betti.html. Clearly, all responsibility for inexactitudes contained in the present paper is mine.

[2] N.B. *not* as a theory of 'general objects', at least in Lesniewski's view.

[3] *Analiza i konstrukcja pojêcia przyczyny* [*An analysis and construction of the concept of cause*], «Przeglad Filozoficzny», 9, (1906), pp. 105-179.

analysis of the relationships among the logical values of propositions; Lukasiewicz calls logic «the science of logical values». Some examples are provided by Lesniewski in order to elucidate the above mentioned distinction; a typical metaphysical expression is:

(i) every object possesses the properties $c_1 \ldots c_n$

Expressions which are *not* metaphysical are:

(ii) some object possesses the properties $c_1 \ldots c_n$
(iii) every object that possesses the properties $c_1 \ldots c_n$ possesses also the properties $d_1 \ldots d_k$

As a matter of fact, (i) states something about *every* object without restrictions, whereas (ii) and (iii) don't. Metaphysics essentially deals with generality. Logic, on the contrary, deals with a certain *kind* of objects: propositions; and logical expressions are assertions which predicate truth and falsity of such objects. Typical logical expressions are:

(iv) The proposition «Paris is in the Principality of Monaco» is false
(v) The proposition written on that wall is true
(vi) Every contradictory proposition is false
(vii) Given two mutually contradictory propositions, if one is false, the other is true

Logical propositions such as (iv) and (v) are said *singular*; they deal with one particular proposition only. Those like (vi) and (vii) are said *general*; they deal with a class of propositions sharing a certain form. Thus, not every proposition is a logical proposition, but we always can *obtain* a logical proposition by asserting of a given proposition that it is true or false. For example, given any (non logical – for example mathematical) true proposition of the form 'A is B' we can obtain the (logical) true proposition "the proposition 'A is B' is true"; and, from this last one, the (logical) true proposition «the proposition "the proposition 'A is B' is true" is true», and so on (the same holds for a false starting point).

One could object that the characterization of logic mentioned above, according to which it is «the discipline that investigates which propositions are true and which are false», entails a reduction of every scientific problem (pertaining to special disciplines) to a logical one. But –as Lesniewski observes- that is not the case. Consider the following problems:

(viii) Is the square constructed on the hypotenuse of a right-angled triangle equal to the sum of the squares constructed on the two catheti?
(ix) Is every body heavy?
(x) Is the theorem of Pythagoras true?
(xi) Is the claim that every body is heavy true?

The questions (viii) and (ix) respectively express a geometrical and a physical problem; on the contrary both (x) and (xi) express logical problems: they're about *propositions* and their being or not being *true*. In particular, (x) does not concern certain relationships among lines, surfaces, and other geometrical entities, rather it concerns the relationship between the proposition expressed by the theorem of Pythagoras (which is *not* a geometrical figure) and truth.

§ 2. The refusal of the Excluded Middle by the early Lukasiewicz

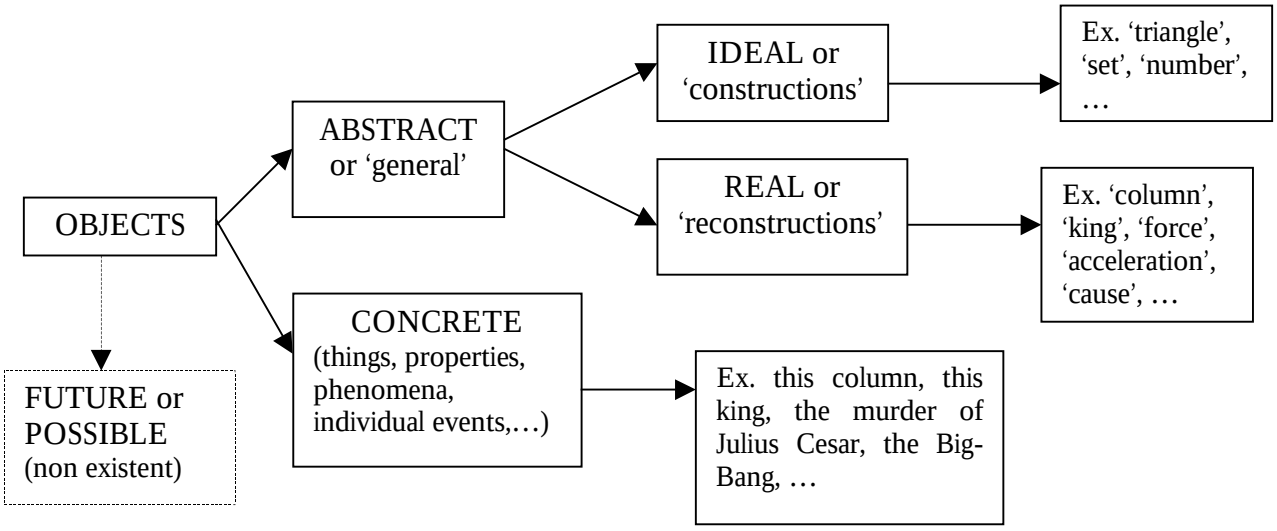
Around 1910 Lukasiewicz accepts –to criticize them- the following formulations[4] of the principle of the Excluded Middle:

[OEM] For any given object *a* and any property *b*, *a* possesses *b* or *a* possesses *non-b*

[LEM] For any two propositions of the form, respectively, «*a* is *b*» and «*a* is *non-b*», they cannot both be false at the same time

The first principle is the *ontological* version of the Excluded Middle; the second is the *logical* version. It does at once results that Lukasiewicz here employs what is commonly called 'term negation' (or 'internal negation') instead of the usual 'sentence negation' (or 'external negation'). This fact is probably to be connected to his previous formation, strongly influenced by an Aristotelian-scholastic approach to the philosophy of logic (*logica terminorum*).[5] According to the term negation approach, the expression of natural language «*a* is not *b*» is to be logically construed as «*a* is *non-b*», contrary to the sentence negation approach, according to which that very expression of natural language is to be construed as «'*a* is *b*' is false». As Lukasiewicz in this context deals with negative properties, he introduces the principle of the Excluded Middle by means of term negation.

Lukasiewicz's refusal of [LEM] is a consequence of his previous criticism of [OEM]: it is *because* the ontological principle does not hold that the logical one doesn't hold either. The criticism of [OEM] is based on the background ontology accepted by Lukasiewicz around 1910, a 'redundant' ontology strongly influenced by Meinong and Twardowski. For the sake of perspicuity, I try below to give a sketch of such a background ontology.



[4] The two principles are not explicitly formulated in these very terms by Lukasiewicz; nevertheless, they can easily be drawn by analogy from his treatment of the principle of contradiction in [1910b] and from his hints to the principle of the Exclude Middle in [1910c].

[5] For a survey on Lukasiewicz's philosophical formation and the intellectual environment in which his early views grew up, see: J. Woleñski, *Logic and Philosophy in the Lvov-Warsaw School*, Kluwer, Dordrecht, 1989 (in particular the 3rd Chapter).

A peculiar feature of abstract (or general) objects (both ideal and real ones) is their *incompleteness*: they are 'incomplete objects', contrary to concrete objects, which result 'complete' under every respect. This last distinction becomes more clear if we assume that an opposition between *essential* properties and *accidental* properties (of a given object) holds. For the triangle in general (the triangle as a general object) it is essential, for instance, to possess three sides, but it's accidental to be equilateral. Analogously, for the king in general it's essential to have a reign, but it's accidental for him to be the king of Spain.[6] Thus, abstract objects come out determinate *only* with respect to their essential properties, whereas they are indeterminate with respect to the accidental ones. On the contrary, concrete objects come out determinate *both* respect their essential properties *and* to the accidental ones; in this sense concrete objects are complete and abstract objects are incomplete. Beside abstract objects, Lukasiewicz seems to allow place for possible and future objects, which are different from nothing, and nevertheless they don't possess existence. Instances of such objects are the future Pope, the next film by Steven Spielberg, the next sea battle in the Persian Gulf. Objects of this kind too are –like the abstract ones- incomplete with respect to many accidental properties (as a matter of fact, the future Pope is at present neither Chinese nor non-Chinese).[7]

In this way, taken any abstract object (both ideal and real), for instance the triangle, it will be neither equilateral nor non-equilateral, as the general triangle is indeterminate both with respect to 'equilaterality' and to 'non-equilaterality'. A similar case have we got if we face the question whether the king in general is Spanish or non-Spanish. But that amounts to say that the principle [OEM] does not hold, generally speaking, for abstract (or future) objects; as there is an entire field of reality to which [OEM] cannot be applied, it is not worth being kept as a genuine metaphysical principle.

In [1910b] Lukasiewicz characterizes in the following way the notion of 'true proposition':[8] «An affirmative proposition I designate as true when it confers on an object the characteristic appropriate to it» (p.494). From the context it is clear that this is not merely a sufficient condition for the truth of a proposition, but a necessary one also. Thus, if we consider the following couples of propositions, it turns out in the light of the preceding remarks that *each one* of them is false:

(xii) the triangle [in general] is equilateral
(xiii) the triangle [in general] is non-equilateral
(xiv) the king [in general] is Spanish
(xv) the king [in general] is non-Spanish
(xvi) the future Pope is Chinese
(xvii) the future Pope is non-Chinese

As a matter of fact we have here an attribution of (positive and negative) properties to objects which cannot possess them, so that such an application of properties is inappropriate. The previous propositions come out thus untrue according to the above characterization; and if we

---

[6] In the same way, for *a* king of Spain it will be essential to be the king of Spain (and to be catholic, …) and other properties will be for him accidental (for instance being blonde).

[7] A lot of problems arise in my view from the kind of ontology assumed by Lukasiewicz. Just to point out one of them, take the number 3; in which box of Lukasiewicz's classification are we allowed to put it? Numbers are ideal objects, but the number 3 seems to be perfectly determinate both respect to its essential properties (as a number) and to its accidental properties (being odd, prime, …). Do we have then to put it among concrete objects? Or do we have to postulate among ideal object a subset of 'ideal-concrete' objects? (…)

[8] He adds that this is the «only […] principle that cannot be demonstrated in terms of other principles but which is rather true and demonstrated "trough itself"» (p.494).

take 'untrue' = 'false' (as we *do* at the moment) the propositions (xii)-(xvii) are false. And this amounts to claim that [LEM] is not a valid logical principle, as it allows exceptions. Note that in the same paper [1910b, p.491] Lukasiewicz explicitly denies that there could be differences in degree between being true and being false.

§ 3. Lesniewski's criticism of 'general objects' and his refusal of the logical principle of the Exclude Middle

Lesniewski's criticism of the logical principle of the Exclude Middle is very different from Lukasiewicz's. His attack on the principle in question is not based on any 'redundant' ontological background, rather on a certain semantic theory. The outcome will be that it is *only* the logical principle of the Exclude Middle (in the formulation given below) that doesn't work; the ontological one is kept as a good ontological principle: reality is perfectly determined between one of two different possibilities.

On the basis of a nominalistic approach, Lesniewski adopts an ontological background that could be said 'sober'. In his view the following expressions are perfectly synonymous each other: «being an object», «being existent», «being something», «being an individual», «being real», «being non contradictory», «to possess space-temporal dimensions». According to this approach, there's only one 'ontological level' and only one kind of existence. On this basis, Lesniewski claims (*versus* Lukasiewicz, Meinong, …) that *no object is a 'general object'*; and he works out in [1913b] an interesting proof in support of this thesis. In the following I'll try to sketch such a proof.

Suppose (*ex absurdo*) that general objects do exist. Let $\mathbf{P_k}$ be one of these objects; it will be defined in terms of a distinctive property which is shared by *all* the corresponding individuals p_1, p_2, …, p_n, … instances of $\mathbf{P_k}$. We have then:

1. for every individual object p_i there is at least one property C_k such that only p_i possesses C_k (such a property amounts –at the very least- to what distinguishes p_1 with respect to the other individual objects).
2. Thus $\mathbf{P_k}$ does not possess C_k (as it possesses only what is common to every individual instance).
3. Let N be 'the property of not possessing the property C_k'. The individual object p_1, then, doesn't possess the property N (otherwise, by 1, it would be a contradictory object).
4. Then $\mathbf{P_k}$ does not possess N either.
5. Thus $\mathbf{P_k}$ does possess C_k.

As there is a contradiction between 2 and 5, the hypothesis concerning the existence of general objects is not tenable. Besides the intrinsic interest this (kind of) argument can have, the moral to be drawn for our present concern is that the logical principle of the Excluded Middle cannot be refused on the basis of a supposed distinction between general (or abstract) objects and individual (or concrete) ones.

By «principle of the Excluded Middle» Lesniewski means the following claims (respectively, ontological and logical version):

[OEM*] Every object must either possess or not possess every property

[LEM*] At least one of the two contradictory propositions has to be true

The first one of these principles is in Lesniewski's view (and contrary to Lukasiewicz) a well-founded metaphysical principle, and as such it has to be kept. The same thing doesn't hold in relation to the second principle; in this connection Lesniewski claims [1913b, p.47]:

> the logical principle of the excluded middle not only does not help to resolve 'logical' problems of various kind, but it is in fact a dangerous theoretical obstacle which should be, therefore, removed from science: it is invalid and, as all invalid principles, should be removed from circulation.

Thus, in order to prove that the logical principle of the Excluded Middle is invalid, one has to find some counterexamples to that principle, pointing out some couples of mutually contradictory propositions none of which is true. To this aim, Lesniewski frames a certain semantic theory concerning the truth-value of propositions containing empty terms; in so doing he introduces the following 'semantical conventions'.

The basic question we have to face in framing such a semantic theory amounts to ask ourselves which is the 'specific function' of the proposition.[9] In Lesniewski's view, this function basically consists in representing a relation of inherence; more precisely he says: «…the proposition can represent only the possession by the object represented by the subject of the proposition, of the properties connoted by its predicate» [1913b, p.56-57]. Thus, when the proposition is true, it represents a real relation of inherence – a fact (or, in Lesniewski's terminology, it has 'symbolic function'); on the contrary, when the proposition is false, it doesn't represent anything (it has not 'symbolic function'). From these remarks, the following claim concerning a necessary condition for the truth of any proposition can be drawn:

(#) if a proposition is true, then it possesses a denoting subject *and* a significant predicate

This means (by opposition) that

(##) if a proposition does not possess a denoting subject *or* a significant predicate, then it is untrue

which amounts to say that it is false, if we assume (as we do) that 'untrue' = 'false'.

Thus, according to (##), if we take any proposition containing an empty subject, neither that proposition nor its negation will come out true, rather they both will be false, contrary to what is maintained in [LEM*]. For instance, all the following mutually contradictory propositions turn out false:

(xviii) every centaur has the tail
(xix) some centaur hasn't the tail
(xx) the smallest rational number grater than 1 is periodical
(xxi) the smallest rational number grater than 1 is not periodical

In this way, the very logical principle of the Excluded Middle turns out to be invalid. Lesniewski proposes a weakened form of this principle, namely:

[LEM**] Given two contradictory propositions, if one is false, then the other is true provided it has a denoting subject and a significant predicate

[9] In the same way in which we assume that the specific function of a proper name is that of denoting an individual object, the specific function of a k-places relation is to denote a set of ordered strings consisting of k individuals, etc.

which he calls «restricted» logical principle of the Excluded Middle.

At this point, it's worth noting that both Lukasiewicz criticism of the Excluded Middle and Lesniewski's, move from the (till now) undisputed acceptance of the following meta-logical principles:

[LV] There are exactly two logical values, 'true' and 'false'

[DET] Every proposition possesses exactly one logical value

from which it follows:

[BIV] Every proposition is either true or false.

That is, both criticisms of the Excluded Middle sketched above start from a *bivalent* basis. As we'll see more closely in the next paragraph, Lukasiewicz only disputes explicitly (after 1917) the principle [BIV], even if sometimes he continues calling it «Excluded Middle». On the contrary, Lesniewski will remain a firm supporter of bivalence and will harshly criticize the philosophical motivations underlying the many-valued logical systems worked out first by Lukasiewicz.

Before giving up bivalence, some remarks concerning the import of Lesniewski's semantics are in order. First of all, if we accept the principle (##), that is, the idea that every proposition containing at least an empty term is false, we cannot maintain –as many authors do- that negative propositions are propositions asserting the falsity of the corresponding affirmative propositions; namely, we cannot accept the theory according to which two propositions of the form, respectively, «*a* is not *b*» and «'*a* is *b*' is false» are *synonymous.* As a matter of fact, if two propositions are synonymous, they must (at least) have the same logical value; but if you take an empty term '*a*', according to (##) the proposition «*a* is not *b*» turns out *false*, as does the corresponding affirmative proposition «*a* is *b*». Consequently, the proposition «'a is b' is false» turns out *true*, and this is a sufficient ground to claim that it's not synonymous with «*a* is not *b*». Thus, it seems that if we accept the semantical principle (##), we're not entitled to construe negative propositions by means of sentence-negation.

Secondly, in the light of (##) we can argue for the falsity of the tenet that every analytical proposition is true. Indeed, the following two propositions:

(xxii) a contradictory object is contradictory
(xxiii) a squared circle is a circle

are both analytical (as the property expressed by the predicate is included in the subject) *and* false, being «contradictory object» and «squared circle» empty terms.

Finally, if we agree with Lesniewski on the invalidity of the logical principle of the Exclude Middle, we are able to criticize a certain kind of argument (also known as «Meinong's paradox») according to which the existence of contradictory objects would turn out of necessity. The sketch of the argument is as follows:

a) no object is a contradictory object; [hypothesis *ex absurdo*]
b) thus, the proposition: «a contradictory object is an object» is false;
c) therefore, the proposition: «a contradictory object is not an object» is true;
d) the proposition quoted in c, being true, is true *of* something;

e) as it is true of something, it has a *denoting* subject, namely the expression: «contradictory object»;
f) thus, a contradictory object *is* an object;
g) we conclude that some object is a contradictory object. [versus the hypothesis][10]

Yet, the inference from b to c *assumes* the validity of the logical principle of the Excluded Middle; but if we stick to (##) both the proposition quoted in b and the proposition quoted in c are false, so that there is *nothing* of which a certain proposition concerning contradictory objects has to be true.

§ 4. Lukasiewicz's refusal of the principle of Bivalence. The third logical value

Around 1910, the refusal of the logical principle of the Excluded Middle by Lukasiewicz and Lesniewski is –as we've seen- entirely internal to the bivalent background. However, around 1917[11] Lukasiewicz disputes the very principle of bivalence [BIV], by criticizing the idea –expressed by [LV]- according to which there are exactly two logical values, 'true' and 'false'.

As far as the *philosophical* motivations are concerned which led Lukasiewicz to work out a three-valued calculus there was doubtlessly (i) the strong link that he saw between the principle of bivalence and the deterministic thesis, together with (ii) his stark aversion for the view of a universe in which every event (even the smallest) turns out to be settled since eternity, and «man's free creativity» comes out definitively compromised. The main contribution in which Lukasiewicz tries to focus the link between bivalence and determinism, and in which he theorizes an overcoming of classical (two-valued) logic is certainly *On determinism* (op. cit.).[12] In what follows I'll try to sketch the main thesis and argumentative strategies emerging from *On determinism*, pointing out some open questions too.

Determinism, as Lukasiewicz takes it, is an essentially *semantical* claim which –if it's true- has immediate metaphysical consequences (one for all, the denial of free will). The deterministic thesis can be stated as follows:

[D] For any given object *x* and property *P*, if *x* is *P* at instant *t*, then, for every instant *s* (in particular: preceding *t*) it is true at instant *s* that *x* is *P* at instant *t*

According to [D] truth is 'eternal', in the sense that if some fact has occurred at a given time, it has been true since eternity to assert that it would have occurred at that time, it is true now to assert it, and it will remain true in the future for all eternity. The real import of this claim can be fully appreciated if we consider sentences in the future tense; take for instance

(*) tomorrow noon Bin Laden will be captured by the American soldiers

---

[10] The underlying structure of this argument is expressible by means of the classical logical law known as *consequentia mirabilis*: $(\neg A \rightarrow A) \rightarrow A$; if a certain claim follows from its denial, than that claim holds.

[11] Lukasiewicz himself informs us about the time of this refusal; see for instance: J. Lukasiewicz, *Farewell Lecture by Professor Jan Lukasiewicz, delivered in the Warsaw University Lecture Hall on March 7, 1918*, in: *Selected Works*, (op. cit.), pp. 84-86. In 1920 appeared the first contribution in which the Polish logician worked out the trivalent system for the propositional calculus; see: J. Lukasiewicz, *On three-valued logic*, in: *Selected Works*, (op. cit.), pp. 87-88.

[12] This paper born as an inaugural address that Lukasiewicz held as rector of the Warsaw University for the inauguration of the academic year 1922-23. During the following years, he worked again on this address several times, improving its form and contents, and aiming to its publication. It was published for the first time in Polish with the title *O determinizmie*, in Jan Lukasiewicz, *Z zagadnieñ logiki i filozofii*, (J. Slupecki ed.), Warsaw, 1961.

uttered, say, today. According to [D], if the fact expressed by (*) occurs, that is, if tomorrow noon Bin Laden is captured by the American soldiers, then it is true at every instant earlier than tomorrow noon to assert that such a fact will occur: it is true now, it was true yesterday, one year ago, … Analogously, if the fact expressed by (*) does not occur, it is true at every instant earlier than tomorrow (now, yesterday, one year ago, …) to assert that such a fact will not occur - that is, it's always been false to assert the proposition expressed by (*). Besides, the principle

[BIV] Every proposition is either true or false

forces us to accept that the proposition expressed by (*) is already *now* determinately either true or false. If it's true, the relevant fact will occur, and if it's false the relevant fact will not occur. But according to [D], both the proposition asserting the occurring of the relevant fact and the proposition denying the occurring of the same fact have got a truth-value which is settled since eternity. Thus, such a fact (and in principle *any* fact) is decided since eternity and therefore it seems as if we're entitled in calling it *necessary*.

But which is (beyond our intuitions) the real relationship between the principle of bivalence and the deterministic claim? In Lukasiewicz's view it is a relation of *logical inference* in the strict sense of the word: for him it is possible to deduce the claim [D] from [BIV] by the application of some simple logical lemmas of propositional calculus, provided an appropriate language is given to express the two thesis in a formal fashion.[13] The problem is that Lukasiewicz only gives an informal argument (in the natural language) for the derivation of [D] from [BIV], without specifying the language in which the proof can be carried out rigorously, but stressing that this is yet possible. Such an informal version of the argument faces, in my view, several interpretative problems and it's not always so clear.[14]

Anyway, the details of the argument mentioned above go beyond the purposes of our present concern. At present it is sufficient to keep in mind that in Lukasiewicz's view a relation of logical inference holds between [BIV] and [D]. Yet, the deterministic claim is a thesis which Lukasiewicz opposes, both from an *ethical* point of view (determinism compels us to deny men's free creative activity and reduces us to mere spectators of the events, the course of which we cannot affect) and from a *theoretical* one (it's somewhat unreasonable that the truth-value of a sentence like «The next year on 14th September at 12.56 o'clock a fly will buzz in my left ear» be already determined now and since eternity!). Therefore, if we want to reject determinism, we have (at least[15]) to show the falsity of the thesis from which it logically follows, namely [BIV].

Now, how is it possible to question the principle of Bivalence? According to Lukasiewicz, a good strategy consists in putting ourselves the following (philosophical) problem: *what is it that makes a given proposition true?* [BIV], as it stands, doesn't give us any hint as to *why* a given proposition ought to be true or false, or as to *what* renders such a proposition true or false. But it is precisely by putting such a question, that it becomes possible to show how [BIV] doesn't have a *general* application (as it admits counterexamples), so that it's not worth being considered a well-

---

[13] Such a language should presumably have, for instance, a truth-predicate (or operator), variables for arbitrary time instants, variables for propositions (or some means to build up the name of any sentence), a certain kind of relation connecting propositions (or sentences) and time, a relation holding between time instants, and so on.

[14] In a previous paper I've tried to reconstruct Lukasiewicz's argument from [BIV] to [D] in a way as linear as possible, providing a language in which the two basic thesis can be formally expressed and the proof carried out. The paper in question, *Logic and determinism in Jan Lukasiewicz's philosophy*, is available for downloading on my web-page http://www.unifi.it/unifi/philos/persone/becchi.htm.

[15] Proving that the principle of bivalence is false does not amount in any way to getting rid of determinism once for all; it merely amounts to showing that one of the classical ways to support determinism is not tenable..

founded logical principle. To this aim, Lukasiewicz declares to accept the following as a valid tenet:

[TM] In order that a given proposition may be true at instant *t*, there must be at instant *t* an actual correlate that makes it true

By «actual correlate» here is meant something (a fact, an event) which is the cause of the truth of the proposition, namely its 'truth-maker'. Thus, if I *now* utter a sentence, it will express a true proposition if and only if there occurs *now* a fact which is the actual correlate of that proposition.

Keeping these remarks in mind, take now the following instance of the principle of Bivalence:

[BIV*] the proposition expressed by the sentence «tomorrow noon Bin Laden will be captured by the American soldiers» (as uttered now) is now either true or false

As we have to evaluate an exclusive disjunction, we face two cases:

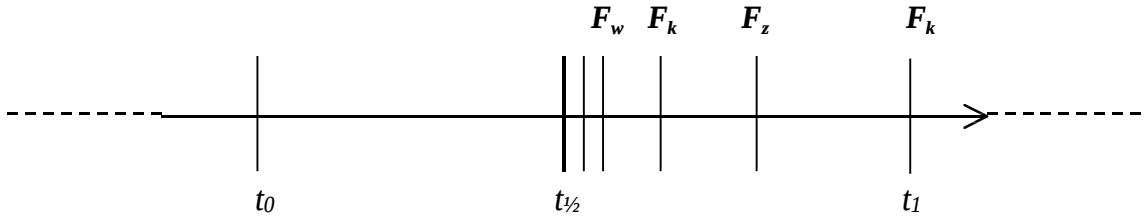
(a) the proposition expressed by the quoted sentence in [BIV*] is now true; then, by [TM] there exists now an actual correlate which is the cause of the truth of the given proposition.
(b) The proposition expressed by the quoted sentence in [BIV*] is now false; that is, it is now true its denial, namely the proposition expressed by the sentence «tomorrow noon Bin Laden will *not* be captured by the American soldiers»; therefore, by [TM] there exists now an actual correlate which is the cause of the truth of the proposition expressed by this last sentence.

Nevertheless, the following considerations show that at present there could not be *neither* a fact making true the given proposition *nor* a fact making it false.

According to a classical line of argument, the deterministic thesis can be drawn from the principle of causality. Lukasiewicz shows that even accepting a strong (metaphysical) version of this principle, one is not compelled to accept determinism; and the way he argues for this last claim represents the very core of his philosophical criticism of [BIV]. By «principle of causality» Lukasiewicz means the following tenet:

[PC] every event has its cause in another event preceding it in time and from which it necessarily follows as effect

This principle, as it stands, seems to entail determinism; according to [PC] *every* event follows necessarily from another event preceding it in time, so that it seems plausible to assert that *at present* (and really since eternity) the causes of *all* future events occur. On the semantical side this means that if we're dealing with future-tensed sentences, by asserting their present truth we are forced by [TM] to admit the present existence of some fact too which is the cause of the future event described by such a sentence. But in Lukasiewicz's view, this is not the case. Indeed, we can coherently maintain that the infinite causal sequence from which a certain future fact F_k necessarily follows has a 'lower limit' which is set at an instant of time *subsequent* the present moment, so that now no causes of F_k exist. How is it possible to maintain that the causal sequence from which F_k originates is nevertheless infinite? This becomes possible if we represent time as a continuous succession of instants standing in a one-to-one relation with the facts belonging to a certain causal sequence, in the following manner:



In the above representation t_0 stands for the present moment, t_1 for the future instant at which the fact F_k occurs and $t_{½}$ for the lower limit of the causal sequence of facts …F_w, …, F_k, …, F_z, …, from which F_k originates. Such a sequence is constituted by *infinite* facts each one taking place at some instant of time t_n, for ½<n<1. Such a sequence hasn't got a beginning or, in other words, there's no a first cause; in fact, such a first cause should occur at the instant of time corresponding to the smallest real number greater than ½, but such a number clearly does not exist. In particular, such a causal sequence, doesn't reach the present instant t_0; and this means that at present does not occur *any* cause of the fact F_k. Therefore, as far as the present instant is concerned, such a fact is wholly indeterminate (it can equally happen or not happen). And this indeterminacy is something *ontological*, not merely epistemic: it depends on the very structure of reality. The main moral to be drawn from these considerations can be expressed in two points: (i) there are infinite causal sequences which belong entirely to the future and none of which has yet begun at present; (ii) the principle of causality in itself, even in a strong form as the one sketched above, does not compel us to accept a deterministic view of the world.

In the light of these remarks, let's go back to the evaluation of the sentence [BIV*]; we have now grounds to affirm that, even if every event is causally determined by some other event preceding it in time, at the present instant does not occur *neither* the cause of Bin Laden's tomorrow capture *nor* the cause of Bin Laden's tomorrow non-capture, because we can coherently assume the existence of a lower limit of the relevant causal sequence placed at some future instant. This means that at present there's not an actual correlate making the sentence quoted in [BIV*] true or false; such a sentence turns out to be at present neither true nor false. Thus we've found a counterexample to the principle of Bivalence and as a matter of fact we can find, by considering future-tensed sentences, infinite counterexamples. Therefore, the principle [BIV] cannot be kept as a valid logical principle.

But do future-tensed sentences have any logical value? If we want –as Lukasiewicz does- to assign a specific logical value to them, we have to reject [LV] and postulate a *third* value; intuitively, this is the value which is assumed by those sentences which are not (yet) true or false.[16] Such sentences describe facts which at present can happen as well as not happen; that is, they describe facts which are now (ontologically) *indeterminate*. The «indeterminate» is therefore the third logical value, beside «true» and «false». Lukasiewicz sometimes calls this third value also «possible», and asserts (in [1918]) that in addition to «being» and «not being» there exists «objective possibility». This is what every indeterminate sentence denotes at present.

Appendix: a proposal concerning the relationship between Excluded Middle and Bivalence

Keeping in mind some concepts and distinctions employed in the preceding paragraphs, and yet adding new ones, in the following I will advance a proposal with the aim to put into a

[16] Even if they presumably will *acquire* the value 'true' or the value 'false' at some instant in the future time.

sharper focus the relationship between the principle of Bivalence and the principle of the Excluded Middle. To this aim, an attempt to (at least roughly) characterize a distinction between logical and metalogical principles seems appropriate.

By *metalogical* principle we mean (in first instance) a claim concerning the logical values which a given proposition can assume; thus, a metalogical principle concerns essentially the quantity and the quality of the very logical values. By *logical* principle we mean a thesis concerning the relationship between the logical values and a given proposition; typically, a logical principle settles a certain relation between the propositions of a given form[17] and such logical values.

According to classical semantics, the logical values a given proposition can assume are two: 'true' and 'false'. As far as their quality is concerned, all that's essential is the disjunction of the two values; that is, what is true is not false and what is false is not true. This is the core of the principle of Bivalence, that can be expressed in a more precise way as follows:

[BIV] If T is the set of true propositions and F is the set of false propositions and *p* is a variable ranging over propositions, then it holds:[18]

$$\forall p\,(p \in V \vee p \in F) \wedge (V \cap F = \Lambda)$$

Clearly, if we take the disjunction as exclusive, we can omit the second member of the conjunction. Note that the principle of Bivalence so formulated embraces in itself the principle of semantical determinateness [DET], according to which every proposition *must* (and not simply *can*) possess exactly one logical value.

On the other hand, we can formulate the principle of the Excluded Middle as follows:

[EM] every sentence of the form $\tilde{N} \vee \neg\tilde{N}$ expresses a proposition which assumes 'true' as its logical value, whichever the logical value assumed by the proposition expressed by $\tilde{N}$ may be.

According to this last principle, then, 'true' necessarily belongs to one of the two members of the contradiction, there's no place for a third possibility. Put in other words, the same principle can be expressed by saying that at least one of two mutually contradictory propositions has to be true (see §3).

It's clear enough that the two principles have to be carefully kept separate: indeed, one can accept [BIV] and reject [EM]. As we've seen in §3, we can adhere to a semantic theory according to which every sentence S (both simple and compound) containing at least an empty term expresses a false proposition; if we do so, given a sentence S containing an empty term, the compound sentence:

(*) $S \vee \neg S$

does not express a true proposition, as both S and $\neg S$ are false; that is, if we accept [BIV], the proposition expressed by (*) is *false*. From this view, [EM] is not a valid logical principle, since it allows counterexamples.

But if it's possible to accept [BIV] and reject [EM], it doesn't seem possible to accept [EM] and reject [BIV]. In fact, in order that one of the two members of the contradiction be true in any case, it is necessary (and sufficient) that the denial of 'false' amount to 'true' (and not, for

[17] Or, more exactly, the propositions expressed by *sentences* sharing the same logical form.

[18] For the sake of the present concern we put aside all possible problems connected with paradoxes arising from the unrestricted use of the expression: «all the propositions».

instance, to a third logical value); that is, it is necessary (i) that the domain constituted by the propositions as a whole be identical to T**N**F and (ii) that that the intersection between T and F be empty. But these two conditions amount exactly to the content expressed by [BIV].

There seem to be good reasons, then, to maintain that [BIV] *is a necessary but not sufficient condition for* [EM], *and* [EM] *is a sufficient but not necessary condition for* [BIV]. As it is a sufficient condition, [EM] is a stronger principle than [BIV]: in fact a sufficient condition, contrary to a merely necessary one, has the 'power' to produce its effect. Indeed, in [EM] some hypothesis essentially concerning the concept of *negation* are made, contrary to [BIV], in which such hypothesis are not present (or, at least, not explicitly).

Last revision: February, 10th, 2003